DR. BABASAHEB AMBEDKAR MARATHWADA UNIVERSITY, AURANGABAD
DEPARTMENT OF MATHEMATICS

M. Phil. Syllabus (W.E.F. JUNE – 2010)

Paper- I – Some Topics in Analysis

Functions from R^m to R^n. Directional and Partial derivatives of a function from R^m to R^n. The linear derivative for the gradient vector. Jacobeans sufficient conditions for the existence of linear derivative. Operations on differentiable functions and the chain rule. The Mean Value Theorem. Continuously differentiable functions. The implicit and inverse functions theorem. Functional dependence. Second and higher order derivatives. Conditions for the existence and equality of mixed partial derivatives. Mxima and Taylor's Theorem.

Integrals in R^N. Curves, Piecewise continuously differentiable curves (or paths), rectifiable curves, length of a path as an integral. Line integrals, Integrals with respect to arc length. The line integral of a gradient. Integration over intervals in R^n – multiple integrals and repeated integrals. Green's theorem for intervals in R^2. Integration over arbitrary bound sets in R^n. The Mean value Theorem for multiple integrals. Differentiation and Integration. Transformation of Integrals. Functions defined by integrals. Green's theorem for regions defined by rectifiable Jordan curves.
(Based on Chapters 7 and 8 [1] and chapters 6,7&10 of [2]

References:

1. J.C. Burkill and H. Burkill: : A second course in Mathematical Analysis. Cambridge University, Press (1970)
2. Tom M. Apostol : Mathematical Analysis, Addison-Wesley (1969)
3. Walter Rudin : Principles of Mathematical Analysis, 3rd Edition (International student ed.) McGraw-Hill Kogakusha Ltd., (1976)

Paper II Algebraic Theory of Semigroups.

Unit- 1

Introductory Ideas: - Basic definitions, Monogenic semigroups, Ordered sets, Semilattices and lattices, Binary relations; Equivalences, Congruences, Free semigroups, Ideals and Rees congruences, Lattices of equivalences and congruences.

Unit- 2

Greens equivalences: - The equivalences L, R ,, H , J and D , The structure of D – classes, Regular D -classes, Regular semigroups.

Unit-3

0-Simple Semigroups: - Simple and 0-simple semigroups; Principal factors, Rees's Theorem, Primitive idempotents.

Unit- 4

Unions of Groups: - Unions of Groups, Semi lattices of groups, Bands.

Unit-5

Inverse Semigroups: - Preliminaries, The natural order relation on an inverse semigroups, Congruences on inverse semigroups, Fundamental inverse semigroups, Anti-uniform semilattices.

Text Book: -

1. Introduction to semigroup Theory by J. M. Howie (Academic press, 1976).

Reference Books: -

1. Semigroups and Combinatorial Applications by Gerard Lallembent (John Willey and Sons, 1979).
2. Algebraic Theory of Semi-groups by A. H. Clifford and G. B. Preston, Vol. 1. (1961) and Vol. 2. (1967) reprinted with corrections 1971 (American Mathematical Society, Mathematical Surveys).
3. Semigroups by E. S. Lyapein (Translations of Mathematical Monographs, Vol. 3. (American Mathematical Society IIIrd Edition 1974 second printing 1978).
4. Introduction to Semigroups by M. Petrich, C. E. Merril Publication Co. Columbus (1973).
5. Completely : 0-Simple Semigroups by K. M. Kapp and Hans Schnerder Benjamin (1969).

Paper – III- Advanced Course in functional Analysis:

Topological Vector Spaces: Characterization of a TVS by fundamental systems of neighborhoods (nbhds) of O. Hausdorff (or separated) TVS. Locally convex spaces (LCS). Characterization of LCS by fundamental systems of nbhds of O (or local bases) d constiting of absorbing, balanced, convex sets, semi-norms, gauge of an absorbing, balanced, convex set. Characterization of a LCS by a family of semi norms. Examples of sequence spaces, including the Kothe sequence space $K^p(a_i)$, function spaces, spaces of bounded linear operators on a Banach space, a Hilbert space, Linear maps, subspaces, quotient spaces, Isomorphism's, strict morphisms, Bounded sets, normality, metrizability, Products and direct sums, Filters cauchy filters, convergent filters, Complete and quasi complete TVS. Completion of a TVS, Frechet spaces, Compact, precompact (or totally bounded) sets in a Hausdorff TVS. Finite dimensional and locally compact Hausdorff TVS. Initial topologies. Firal topologies

Duality: The Hahn-Banach theorem-geometric and analytical form. Pairings – dual system of vector spaces, weak topology defined by a pairing. Polarity-theorem of bipolar. C Topologies or the topology of uniform convergence on sets belonging to C) ^{c} – topologies compatible with a pairing, equicontinuous sets of linear mappings, Alaoghi – Bourbaki theorem, the strong topology (or the topology of uniform convergencey on weakly bounded sets. The Mackey topology, barels, bornivious sets in a TVS, the Mackey theorem. Barrelled spaces-uniform bounded ness principle, quasi-barrelld (or infra barreled) spaces. Boronological spaces. Reflexivity-semi-reflexive and reflexive spaces, Banach Bourbaki theorem, Montel (or perfect) spaces- the topology, on the dual space E', of uniform convergence on balanced convex, compact subsets of a Hausdorff LCSE, Montel's theorem on normal families of analytic (or hope orphic) functions. Banach Dieudonne and Krein-smulian theorems. The transpose (or adjoint) of a linear mapping. Duals of subspaces and quotient spaces. Duals of products and direct sums. The homomorphism theorem and the closed-graph theorem.

The syllabus is based on Cha. 2 and Cha. 3.1 to 3.9 of [1] and Sections3.19 to 3.22, 5.11 of [2].

References Book:

1. John Horvath : Topological Vector spaces and distributions, Vol. 1 Addison-Wesley Publishing Company (1966)
2. Walter Rudin : Functional Analysis. Tata-McGraw- Hill publishing company (1973)
3. Sterling K.Berbarian: Lectures in Functional Analysis and Operator Theory. Springer Verlag (1973)
4. Gottfried Kolthe : Topological Vector Spaces, Vol 1 Springer –Verlag (1973)
5. Francois Treves : Topological Vector Spaces Distributins, and Kernels,Academic Press (1964)
6. Helmut H. Scheefer : Topological Vector Spaces Springer Verlat (1971)

Paper – IV – Theory of Partial Differential Equations

Elliptic Equations: [1]

The Laplace Operator, Second Order elliptic Operators, Transformation. The Maximum Principle of E. Hopf; Uniqueness Theorems for Boundary Value Problems, The Generalized Maximum Principle, Derivative of Harmonic functions, Boundary Estimates for the Derivatives, Applications of Bounds for Derivatives, Nonlinear Operators.

Parabolic Equations: [1]

The Heat Equation, the one –dimensional Parabolic Operator, The General Parabolic Operator, Uniqueness Theorems for Boundary Value Problems, Nonlinear Operators Weakly Coupled Parabolic Systems.

Monotone Method for Elliptic boundary Value Problems: [2]

Review of Linear Elliptic Problems, Positivity Lemma, Method of Upper and Lower Solutions, Existence-Uniqueness Comparison Theorems, Applications.

Monotone Method for Parabolic Boundary Value Problems: [2]

Review of the Linear Parabolic Problems, Positivity Lemma, Method of Upper and Lower Solutions Existence Comparison Theorems, Uniqueness Theorems, Positivity and Boundedness of Solutions

Text Books:

1. M. H. Protter and: H. F. Weinberger — Maximum Principles in Differential Equations, Springer Verlag, New York, 1984.

2. C. V. Pao : Nonlinear Parabolic and Elliptic Equations, Plenum Press, New York, London, 1992.

Reference Books:

1. W. Walter : Differential and Integral Inequalities Springer Verlag 1964

2. L. C. Evans : Partial Differential Equations, Graduate Studies in Mathematics Vol. 19. AMS, 1998

3. D. Gilbarg and: N. S. Trundinger — Elliptic Partial Differential Equations of Second Order, 2nd Edition, Springer Verlag, New York, 1998

Paper - V - Graph Theory

Fundamental concepts:

Definitions an examples, graphs as models, matrices and isomorphism, paths, connected graphs, bipartite graphs, externality vertex degree, the Pigeonhole principal, Turan's theorem, degree sequences. Graphic sequences. Degree and digraphs

Tree and Distances:

Properties of tree, distance in graphs, stronger results, disjoint spanning trees, shortest paths, tress in computer science, Eulerian circuits.

Matching and Factors:

Matching in bipartite graphs, maximum matching, Hall's matching conditions, Mismatching in bipartite graphs, sets, applications and algorithms, maximum bipartite matching, weighted bipartite matching, in general graphs, Tutte's 1- factor theorem, f- factors of graphs.

Connectivity and Paths:

Cuts connectivity, edge-connectivity, blocks, 2-connected graphs, connectivity of digraphs, k connected and k-edge connected graphs, applications of merger's theorem, Network flow problems, maximum network flow, integral flows.

Edges and cycles:

Line graph and edge coloring, Hamiltonian cycles: necessary conditions, sufficient conditions.

Text Book:

Douglas B. West, Introduction to Graph Theory Prentice-Hall, New Delhi (1999).

Reference Books:

1) F. Harary, Graph Theory, Nsrosa, New Delhi.
2) Narsing Deo, Graph Theory, Prentice Hall, India.

Paper – VI : Wavelets, Fractional Calculus and Applications

Part – I: Wavelets Analysis

Brief history, Motivation, Linear algebra, Inner product, Function spaces, Measure and Integration.

Fourier Analysis: Fourier series, Fourier integral, Fourier Transform(FT), existence and inversion, Properties of FT and examples, Convolutions and delta function, Parseval's theorem, Riemann-Lebesgue lemma, Poisson summation formula, Uncertainty principle, convergence theory and signal processing.

Windowed Fourier transform(WFT), types of windows, time frequency localization, Reconstruction formula, Gabor transform and its inversion, Short Time Fourier Transform(STFT).

Continuous Wavelet Transform: Bandlimited functions and Shannon's theorem , wavelets and examples, Admissibility condition, Shifting, Dilation and Modulation operations, Properties of CWT, Parseval's theorem, Reconstruction formula, Time-frequency analysis, Construction of Wavelets, Orthogonal scaling functions, Biorthogonal wavelets, CWT in higher dimensions.

Discrete Wavelet Transform (DWT): Wavelet series, Time-frequency plot, Multi Resolution Analysis(MRA). Refinement relations, Refinement coefficients and their properties, Decomposition, Bases for V_m , Orthonormality and construction of mother wavelets, regularity and moments.

Part – 2: Fractional Calculus:

Special functions of the fractional calculus: Gamma function, Mittag-Leffler function, Wright function.

Fractional Derivatives and Integrals: Grunwald_Letnikov Fractional Derivatives, Riemann-Liouville Fractional Derivatives, Caputo's Fractional Derivatives, Sequential Fractional Derivatives, Left and Right Fractional Derivatives, Properties of Fractional Derivatives, Laplace transform of Fractional Derivatives, Fourier Transform of Fractional Derivatives, Mellin Transform of Fractional Derivatives.

Existence And Uniqueness Theorems: Linear Fractional Differential Equations, General form of Fractional Differential Equations, Method of solutions, Dependence on initial conditions.

The Laplace Transform Method: Standaed Fractional Differential Equations, Sequential Fractional Differential Equations.

Books:

1. Gerald Kaiser, ***A Friendly Guide to Wavelets***, Birkhauser, Boston, 1994.
2. Ingrid Daubechies, ***Ten Lectures on Wavelets***, SIAM, Philadelphia, 1992.
3. Charles K. Chui, ***An Introduction to Wavelets***, Academic Press, Boston,1992.
4. Jaideva C. Goswami, and Andrew K. Chan, *Fundamentals* ***of Wavelets: Theory, Algorithms and Applicatons***, John Wiley & Sons, New York, 1999.