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AURANGABAD**



DEPARTMENT OF MATHEMATICS

Syllabus for PET- 2021

Abstract Algebra

Definition of group, some examples of groups, some group properties, subgroups, cyclic group, cosets, Euler's theorem, Fermat theorem, Lagrange's theorem, order of generator of cyclic group.

Normal subgroups and quotient groups, group homomorphism, isomorphism, fundamental theorem of group homomorphism, automorphism.

Permutation groups, another counting principle, Sylow's theorem, direct product, finite abelian groups, Cauchy theorem.

Definition and examples of rings, some special classes of rings, Homomorphisms, Characteristic of a ring, ideals, subrings.

Quotient rings, Maximal ideals, Homomorphism, the field of quotients of an integral domain, Euclidean rings and polynomial rings.

Linear Algebra

Vector space, subspace, linear span, sum of subspaces, direct sum of subspaces, internal direct sum of subspaces, quotient space.

Linear dependence, independence, bases, dimension, co-ordinate vector relative to basis, dual space.

The inverse of a matrix, application of the inverse of a matrix to a system of linear equations, the rank of a matrix, row rank and column rank, linear equations, homogeneous equations, non-homogeneous equations.

The algebra of linear transformation, range and kernel of LT, isomorphism, First fundamental theorem on homomorphism, internal direct sum, external direct sum.

Matrix and linear transformation, Eigen values, Eigen vector, Cayley-Hamilton theorem, inner product space, orthogonal vectors, (Gram-Schmidt orthogonal process).

Real Analysis

Definition and existence of Riemann-Stieltjes integral, Properties of the integral, Integration and Differentiation, The fundamental theorem of calculus, Examples.

integration of vector valued functions. Rectifiable curve. Examples. Sequences and series of functions. Point wise and uniform convergence. Cauchy criterion for uniform convergence. Weierstrass M-test, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration. Examples.

Uniform convergence and Differential, The Stone – Weierstrass theorem, Examples. Power series, Abel's and Taylor's theorems, Uniqueness theorem for power series. Examples.

Functions of several variables, Linear transformations, Derivatives in an open subset of \mathbb{R}^n , Chain rule, Examples

Partial derivations. Interchange of the order of differentiation, The inverse function theorem, The implicit function theorem Jacobians, Derivatives of higher order, Differentiation of integrals. Examples,

Measure on the real line. Lebesgue outer measure, measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Examples.

integration of functions of a Real variable. Integration of a simple function. Integration of non-negative functions. The general integral. Integration of series. Examples.

Riemann and Lebesgue Integrals, Differentiation. The four derivatives, Functions of bounded variations. Lebesgue's differentiation theorem, Examples.

Abstract Measure spaces. Measures and outer measures Extension of a measure. Uniqueness of the extension. Completion of a measure spaces. Integration with respect to a measure. Examples.

The L^p spaces. Convex functions. Jensen's inequality. The inequalities of Holder and Minkowski Completeness of $L^p(\mu)$ Convergence in measure. Almost uniform convergence. Examples.

Topology

Recall definitions of functions, images and inverse images of sets under given mappings, metric spaces, open disks in metric spaces, open and closed sets and their properties, continuity and its formulations in terms of open and closed sets.

Introduction to topology and topological spaces, open sets, closed sets, closure, interior, neighborhoods, neighborhood systems, neighborhood bases at a point, weaker and stronger topologies, the Hausdorff Criterion, cluster point, derived set.

Base for a topology, sub base for a topology, criterion for base, subspace of a topological spaces, nature of open sets, closed sets, Neighborhoods in subspaces, continuous functions on topological spaces and criteria of continuity, homeomorphism.

Product spaces, box topology on finite Cartesian product, Tychonoff topology or product topology on a general product, evaluation maps, quotient topology, quotient spaces, sequences in topological spaces and their inadequacy.

Net, convergence of net, cluster point of a net, subnet, continuity of functions in terms of net, ultra net, Filters and their convergence, continuity of functions in terms of filters, ultra filter, relation between nets and filters.

Separation axioms T_0, T_1, T_2 , regularity and complete regularity, T_3 and Tychonoff spaces.

Normality and its criteria, T_4 -spaces, Urysohn's lemma, Countability axioms, first countability, second countability, Lindelof spaces, separable spaces

Compactness and its characterizations, compactness together with Hausdorffness and regularity, locally compact spaces, compactification of topological spaces.

Refinement, star-refinement, barycentric refinement, locally finite collection, point-finite collection, paracompact spaces, metrisation of topological spaces.

Disconnected spaces, connected spaces, mutually separated sets and criterion of connectedness in terms of them, components, simple chain, Path wise connected spaces, arc wise connected spaces, locally connected spaces.

Complex Analysis

The Complex number system: The field of complex numbers, The complex plane, Rectangular and polar representation of complex numbers; Intrinsic function on the complex field; The Complex plane.

Metric spaces and Topology of C: Definition and examples of metric spaces; connectedness; sequence and completeness; compactness; continuity; Uniform convergence.

Elementary properties and examples of Analytic functions:

Power series; The exponential function; Trigonometric and hyperbolic functions; Argument of nonzero complex number; Roots of unity; Branch of logarithm function. Analytic functions; Cauchy Riemann Equations; Harmonic function;

Analytic functions as a mapping; Mobius transformations; linear transformations; The point at infinity; Bilinear transformations, Complex Integration: power series representation of analytic functions; zeros of an analytic function.

The index of a closed curve; Cauchy's theorem and integral formula; Gaussat's Theorem; Singularities: Classification of singularities; Residues; The argument principle.

Compactness and convergence in the space of Analytic functions:

Spaces of analytic functions; The Weierstrass factorization theorem; factorization of the sine function; The gamma function; The Riemann zeta function.

Harmonic functions: Basic properties of Harmonic functions and comparison with analytic function; Harmonic functions on a disk; Poisson integral formula; positive harmonic functions

Entire functions; Jensen's formula; The Poisson-Jensen formula; The genus and order of an entire function. Hadamard factorization Theorem.

Univalent functions; the class S; the class T; Bieberbach conjecture; sub class of S.

Analytic continuation: Basic concepts; special functions.

Differential Equations

Existence, uniqueness and Continuation of solutions: Introduction, Method of successive approximations for the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$, The Lipschitz condition. Peano's existence theorem, maximal and minimal solutions.

Continuation of solutions, Existence theorems for system of differential equations: Picard-Lindelof theorem, Peano's existence theorem, Dini's derivatives, differential inequalities.

Integral Inequalities: Gronwall-Reid-Bellman inequality and its generalization, Applications: Ziebur's theorem, Peron's criterion, Kamke's uniqueness theorem.

Linear systems: Introduction, superposition principle, preliminaries and Basic results, Properties of linear homogeneous system, Theorems on existence of a fundamental system of solutions of first order linear homogeneous system, Abel-Liouville formula.

Adjoin system, Periodic linear system, Floquet's theorem and its consequences, Applications, Inhomogeneous linear systems, applications.

Preliminaries, Basic Facts: Superposition principles, Lagrange Identity, Green's formula, variation of constants, Liouville substitution, Riccati equations, Pfaffian Transformation. Higher order linear equations.

Maximum Principles and their extensions, Generalized maximum principles, initial value problems, boundary value problems.

Theorems of Sturm; Sturm's first comparison theorem, Sturm's separation theorem, Sturm's second comparison theorem.

Sturm-Liouville boundary Value Problems: definition, eigenvalues, eigenfunctions, orthogonality.

Number of zeros, Non oscillatory equations and principal solutions, Nonoscillation theorems.

Functional Analysis

Definition of normed linear spaces, Banach spaces, continuity of norm, joint continuity of vector addition and scalar multiplication in normed linear spaces, quotient spaces.

Continuous linear transformations and different criteria of continuity of linear transformations on normed linear spaces, space of bounded linear transformations, isometric isomorphism, equivalent norms, Conjugate spaces, Hahn-Banach theorem and its consequences, natural imbedding of normed linear space into its second conjugate.

The Open Mapping theorem, projections on Banach spaces, the Closed graph theorem, the Uniform Boundedness theorem, conjugate of an operator, Inner product spaces, Schwarz's inequality, joint continuity of an inner product, parallelogram law in inner product spaces.

Hilbert spaces, Orthogonal complements, Orthonormal sets, Bessel's inequality, conjugate space of a Hilbert space, adjoint of an operator, self-adjoint operators, normal and unitary operators.

Partial Differential Equations

First order partial differential equation, linear equations of the first order, integral surface passing through a curve, surfaces orthogonal to a given system of surfaces.

Non-linear partial differential equations of the first order, Cauchy's method of characteristics, compatible system of first order equations (condition of compatibility), Charpit's method.

Special types of first order equations, solutions satisfying given conditions,

a) Integral surface through a curve.(b) Derivation of one complete integral from another.

(c) Integral surfaces circumscribing a given surface. Jacobi's method for solving $F(x, y, z, p, q) = 0$.

The origin of second order equations, linear partial differential equations with constant coefficients, intermediate integrals or first integrals, Monge's method of integrating $Rr + Ss + Tt = V$, classification of second order partial differential equation (Canonical form).

Linear Integral Equations

Definition of Integral Equations and Linear Integral Equations, Types of Linear Integral Equations, Special kinds of Kernels: Separable or degenerate kernel, symmetric kernel, convolution-type kernels, Eigenvalues and Eigen functions of kernels, Solution of linear integral equations, Verification of solution of linear integral equations, Conversion of Boundary Value Problem to integral equations and vice-versa, conversion of Initial Value Problems to integral equations and vice-versa.

Methods of obtaining solution for Fredholm integral equations, Fredholm integral equations with separable kernels, Approximating kernels by separable kernels, Method of successive approximation, Iterated kernel method for Fredholm integral equations, Resolvent kernels and their properties, Methods of solutions for Volterra integral equations, Volterra type kernel, Method of differentiation, Method of successive approximations, Method of iterative kernels, Resolvent kernels and its use to solve Volterra integral equations.

Symmetric kernel, trace of a kernel, Fredholm operator, Fundamental properties of symmetric kernels, Eigen values and Eigen functions of symmetric kernel and their properties, normalized Eigen functions, Iterated kernel of symmetric kernels and their properties, Truncated kernel of symmetric kernel and necessary and sufficient condition for symmetric kernel to be separable, The Hilbert-Schmidt theorem, Method of Solution for Integral equations with symmetric kernels.

Integral Transform Methods, Recall of Laplace and Fourier Transforms, Application of Laplace transform to Volterra integral equations with convolution-type kernel and examples, Application of Fourier transform to some singular integral equations and examples.

Mechanics

Mechanics of system of particles, generalized coordinates, Holonomic & nonholonomic system, Scleronomic & Rheonomic system, D' Alembert's principle and Lagrange's equation of motion, different forms of Lagrange's equation, Generalized potential, conservative fields and its energy equation, Application of Lagrange's formulation.

Functional, Linear functional, Fundamental lemma of Calculus of Variations simple variational problems, The variation of functional, the extremum of functional, necessary condition for extreme, Euler's equation, Euler's equation of several variables, invariance of Euler's equation, Motivating problems of calculus of variation, Shortest distance, Minimum surface of revolution, Brachistochrone Problem, Isoperimetric problem, Geodesic.

The fixed end point problem for 'n' unknown functions, variational problems in parametric form, Generalization of Euler's equation to (i) 'n' dependent functions (ii) higher order derivatives. Variational problems with subsidiary conditions,

Hamilton's principle, Hamilton's canonical equations, Lagrange's equation from Hamilton's principle Extension of Hamilton's Principle to nonholonomic systems, Application of Hamilton's formulation (Hamiltonian) cyclic coordinates & conservation theorems, Routh's procedure, Hamilton's equations from variational principle, The principle of least action. Kepler's law of planetary motion.